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# Role of Figures in Mathematics Problems in Slovak Testing T9

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**Abstract**. Besides providing information to pupils, their parents, teachers and school founders about the achieved level in mathematics, the pupils' results in mathematics at international or national testing can also be used for other purposes. In our research, the results of Slovak national testing T9 (success rate of pupils and difficulty of individual thematic areas and test items) seem to us to be a reasonable source for identification of critical areas in school mathematics. Based on the findings of such areas, we target more at these areas in the preparation of future teachers of mathematics. The special group of problems, so-called problems with figures, seems to be one of the critical areas. In the assignment of these problems, a part of the input information is not of a purely textual character and in the process of solving the solver has to read information about objects appearing in the problem and relations between objects from figures (e. g. scheme, graph, chart, table, picture or map). The paper focuses on success rates of pupils in solving problems of this type and on various roles and functions of figures in problems with figures from the testing T9.

**Keywords**. Critical areas in school mathematics, problems with figures, roles of figures

## 1. Introduction

Testing and evaluation of the pupils' level of knowledge has an undisputable place within the education process. It provides information not only for pupils and their parents, but also for teachers, who can, based on the pupils' results, evaluate the effectiveness of the used educational methods. Besides the most well-known international testing frameworks like PISA and TIMSS also national-level testing is very important. In Slovakia one such nationwide testing framework for mathematics is T9 testing. Its results, spanning over many years, seem to us to be a reasonable source for identification of the critical areas of school mathematics[1].

In this paper, we present partial results of the analyses, which allowed us to set out one such critical area in school mathematics, namely problems with figures.

[1] As critical areas of school mathematics we understand "areas, in which pupils often and repeatedly fail, in other words areas that they do not master or understand enough for their mathematic literacy to evolve and also for them to use it creatively in everyday life." (Rendl & Vondrová et al., 2013: p. 7–8) Identification of such critical areas of school mathematics and consecutive strengthening of the teachers' competence levels regarding these areas may contribute to the improvement of the pupils' mathematical literacy.

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## 2. Slovak testing T9

Slovak nationwide testing T9 is designed for 15-years-old pupils at the end of lower secondary education in Slovak school system (the 9th grade) and is regularly carried out by the National Institute for Certified Educational Measurements (in Slovakia, the abbreviation NUCEM is used) from 2005. The main official aim of this testing is to objectively compare the performance of individual pupils in the subjects selected for testing (Mathematics and Slovak language and literature). Moreover, based on the results, the next aim is to obtain a general view of the performance of pupils at the exit of the second level of education (ISCED 2) and monitor the level of the readiness of pupils for further study (ISCED 3).

As a basis for the quantitative analysis for our research, we used the evaluation of the results of almost 280 000 pupils in all items of T9 testing in the monitored time frame 2012–2018.[2] The statistical analysis was carried out in statistical programme IBM SPSS and the charts in Fig. 1 and Fig. 2 in the paper are the outputs of this programme.

Each year in the monitored time frame the test in T9 consisted of 20 items, 10 of them are open, the other 10 items were closed (choice of one from four options).[3] Every correct answer was awarded 1 point, every missing or incorrect answer was worth 0 points. Difficulty of individual items was determined according to the revised Bloom's taxonomy of educational objectives, from the level of expertise *to understand* up to the level *to evaluate*. Items targeting the lowest and topmost levels of *to remember* and *to create* were not present in the test. Individual test items are divided into four thematic areas:

O1 *Numbers, variables and arithmetic operations with numbers,*

O2 *Relations, functions, tables, charts,*

O3 *Geometry and measurement,*

O4 *Combinatorics, probability, statistics, logic, reasoning, evidence*.[4]

Comparison of the pupils' success rate within individual thematic areas is interesting. The graph in Fig. 1 shows noticeable differences in average success rates across the thematic areas. During the seven yearlong monitoring, the low success rate in the area Geometry and measurement was statistically significant, showing that it could be added to the critical areas in school mathematics, being also in accordance with the results of research *i.e.* in Czech Republic (Rendl & Vondrová et al., 2013). But this thematic area included also problems with average or lower difficulty. It was therefore needed to specify and examine problematic items with high difficulty. A detailed analysis showed, that items with lower difficulty within this area tested procedural knowledge more in arithmetic than geometry.[5] On the other hand, we found "the art of seeing"[6], better known from geometry, also in test items from other thematic areas. From this point of view also those problems, which do not have a strictly textual assignment, are interesting. Sometimes these are called problems with discontinuous text, where a part of or even all-important input data in assignment of problem or data in options (in case of closed items) are graphically visualised by means of figures. These can be of various type – scheme,

[2] In 2020 and 2021, testing T9 did not take place due to the COVID-19 pandemic.
[3] In 2019, the number of test items was increased to 30 and the test time also increased.
[4] In fact, the test items are divided into five thematic areas. The area Logic, reasoning, evidence is separated from the fourth thematic area (Combinatorics, probability, statistics), but the number of test items from this area is low, so it is more suitable to combine these areas in some analyses.
[5] As an example of the "arithmetic" problem in thematic area Geometry and measurement, we present the open question (test item No. 14/2014): *How many times larger volume does a 9 dm sided cube have compared to a 9 cm sided cube? A) 10 000-times, B) 1 000-times, C) 100-times, D) 10-times.*
[6] According to F. Kuřina (1990) "the art of seeing" entails geometric expression, geometric imagination and development of intuition in solving assignments of problems.

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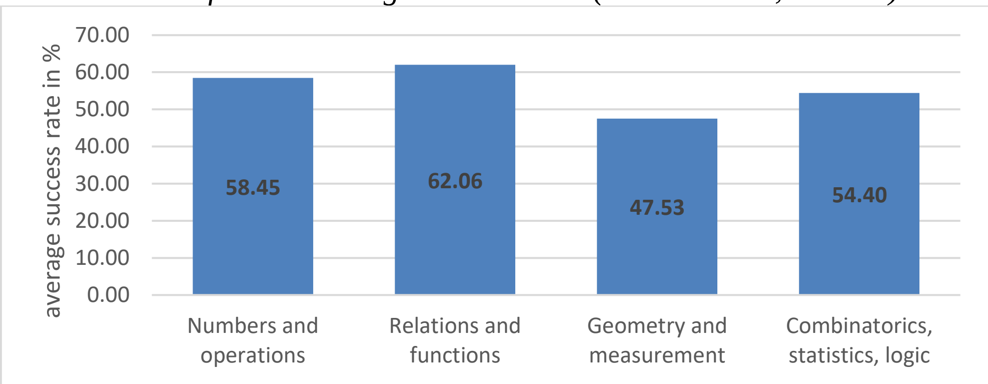
graph, chart, table, picture or map. The solver must visually "read" and process the data in order to solve the problem. We named such problems as *problems with figures* in the article (Csachová, Gunčaga & Jurečková, 2017). The term "figures" was also used by A. L. Mesquita in her article (Mesquita, 1998) as a synonym for representation of a term or situation in geometry.

*Figure 1: Chart showing comparison of success rates in test items in individual thematic areas of the T9 testing in 2012–2018 (source: SPSS, authors)*




**3. Problems with Figures**

Besides the term we introduced – *problems with figures* – there are also others in the literature. For example, Divišová uses the term *problems by insight* (Divišová, 2012), although she does not include graphs or tables, only images used in geometrical problems. In the article (Trahorsch & Bláha & Janko, 2018) the authors warn about the ambiguity and varying interpretation in naming the non-textual elements in textbooks. Besides the term *visual* they also list other terms used in various sources like *nonverbal component, pictorial material, visual element, illustration* or *picture.* However, in our research it's important to state the fact that in school mathematics, assignments of mathematical problems may comprise of more elements than just text.

One of the mathematical competences of the graduates of primary/lower secondary school is also comprehension of text, which contains tables, images, graphs and their interpretation, or further processing of data gathered from them, but also graphical presentation of certain data. Reading or gathering data from such figures is nowadays understood as one of further literacies (Lowrie & Diezmann, 2007). According to the Innovative national education programme, reading tables, graphs and creating them is one of the aims of mathematics as soon as in primary education. Performance standards include requirements like creating various buildings from cubes from a plan and drawing a plan of a block building or propaedeutically depict a given part of the whole (a half, a third, a quarter...) in an appropriate geometric model. In secondary education the requirements for working with tables or graphs is commonplace. Therefore, problems with figures are included in the T9 testing every year.

Most of the problems with figures occurring in T9 testing in the monitored time frame concerned with the following topics of school mathematics and their objectives: *angles and their measurements, circuit and area of planar objects, surface and volume of spatial objects, position of numbers on the number axis, part of a whole as a fraction or a percentage and vice versa, reading data from charts and determination of some numerical characteristics.*

In studying *problems with figures*, we were interested in the following research questions:

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1. *What was the pupils' success rate of problems with figures in the Slovak nation-wide T9 testing in the longer time period?*
2. *What kind of problems with figures proved difficult for the pupils?*
3. *What was the role of figures in problems with figures in school mathematics?*
4. *Could the role of figure in the assignment of problems with figures have influenced the solution of a pupil?*

We have covered the first two questions in the greater detail in the paper (Csachová, Jurečková, 2018), therefore we will present only basic information for them in section 3.1. This paper focuses on research questions 3 and 4, primarily the role of figures and their potential impact on the pupil's solution.

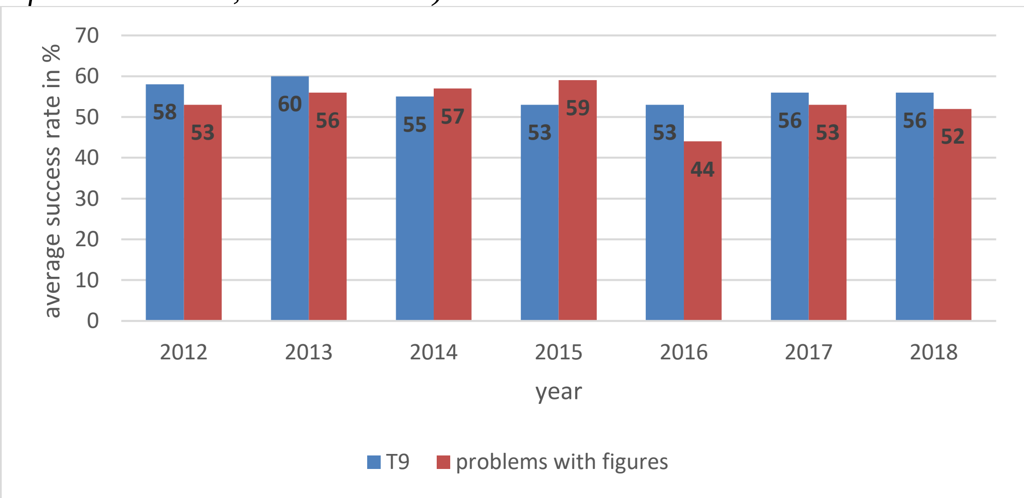
### 3.1 Quantitative analysis of problems with figures

As we stated earlier, problems with figures in T9 testing do not occur exclusively in the thematic area Geometry and measurement, but also in other areas, as shown on Fig. 3, 4, 5. It's important to realize the importance of reading with comprehension in solving problems with figures, as data from the textual assignment often need to be combined with data drawn in the figures.

The total number of problems with figures in T9 tests annually was not stable; in the monitored time frame 2012–2018 it had been between 5 and 12 of the total 20 test items.

Chart in Fig. 2 shows the comparison of general success rate with the success rate in solving problems with figures in T9 tests in the individual year of the monitored time frame. From the perspective of qualitative analysis, year 2015 is interesting, with success rate in problems with pictures was 6% higher than average, as well as year 2016, where it was 9% lower than the general average.

*Figure 2: General average success rate of solving test items in T9 tests compared to success rate in solving test items – problems with figures (in %) in the time frame 2012–2018*[7] *(Source of data: NUCEM, chart: authors)*




[7] The similar chart for the time frame 2012–2017 was published in the article (Csachová, Jurečková, 2018).

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### 3.2 Qualitative analyses of problems with figures and role of figures

In some mathematics problems, the figure featured in the problem's assignment does not contain any "new" data needed to solve the problem (as it is shown for example in Fig. 3). But it seems, such a figure can help to solve the problem. In other problems, the figure in the assignment contains data, without which the solution is not possible, for example, see Fig. 4. In both given examples, the figure is a *picture* in the true sense of the word. Fig. 5 shows a table of students in two classes divided into groups by their final marks, which was used in a problem in the area *Combinatorics, probability, statistics, logic, reasoning, evidence*. It's not a figure per se, but to extract data from it, the pupil needs to "read" a combined assignment, not only a textual part. This requires the pupil to use visual cognitive processes much the same as, for example, a scheme.

"Reading" data off of the figures in the assignment is important mainly for getting the insight of the assignment. This is, for example in (Novotná, 2004) a complex operation comprising of 5 activities: *identification of objects, identification of relationships between objects, identification of the question, finding the uniforming perspective, obtaining insight into the assignment's structure and creation of the mathematical model*. The solution process does not need to include all five of the activities. The first two activities are important for our research, namely identification of objects in the figure and relationships between them, but identification of the question may also be of interest, like in Fig. 6. In this figure, we can identify objects, which hold the data needed to solve the problem, as well as relationships between them. Without reading the textual part of the assignment it is even possible to identify the question, namely what it is that needs to be calculated. But identification of the question right from the figure also often becomes an impediment in the solving process, for instance when a figure is often used in a certain type of assignment, and the pupil's repeated experience prevents them to use the correct procedure in a different situation. See the problem on Fig. 7a for example.[8]

*Figure 3: Test item 5/2012: A block of flats consists of three houses with directly subsequent odd house numbers. The sum of the numbers of the two outer houses is 50. What is the highest house number? (Source: NUCEM, 2012)*

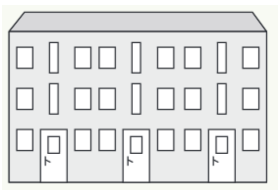
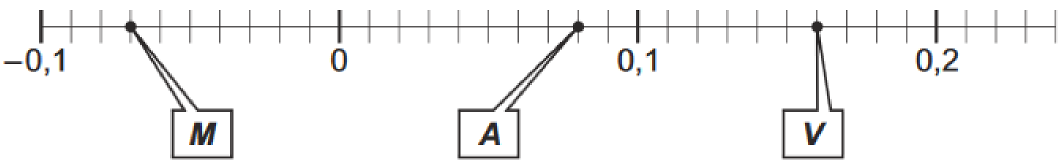


*Figure 4: Test item 2/2017: Numbers M, A, V are marked on the number axis. Calculate M + A + V. (Source: NUCEM, 2017a)*

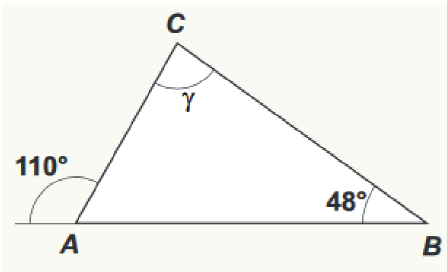



[8] This problem may also occur in the test item from Fig. 11, because the question may not be to determine the dimension of γ, but could well ask for a difference in the dimensions of γ and the angle by the vertex *B*.

*Figure 5: Test item 10/2014: The table contains information about the numbers of students with the same final grade in mathematics in two classes. Based on the information in the table, calculate the average grade in mathematics in class 8.E in the second semester. Enter the result as a decimal number with accuracy of two decimal places. (Source: NUCEM, 2014)*

| TRIEDA | Polrok | Počet žiakov podľa známky z matematiky | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| 8. E | prvý | 5 | 6 | 5 | 7 | 2 |
| | druhý | 4 | 6 | 8 | 6 | 1 |
| 9. E | prvý | 6 | 7 | 6 | 5 | 1 |
| | druhý | 8 | 7 | 8 | 1 | 1 |

*Figure 6: Test item 3/2012: Calculate the dimension of the inner angle $\gamma$ in the triangle ABC on the picture. Enter the dimension in degrees. (Picture for illustration only.) (Source: NUCEM, 2012)*




Problems on Fig. 7a, b represent common problems that ask for calculating the surface area of a planar geometrical shape entered in a square grid. If the pupil would not read the textual part of the assignment, and would identify the question in the first case (Fig. 6a) as determining the surface area of the triangle in the picture, he could then calculate the result in square millimetres. We presume this might have occurred in the testing, as in the T9 testing only 22.7% of pupils got the correct solution and as many as 23 % of pupils did not try to solve the problem at all. In the second case (Fig. 7b) this would not happen, as the measured square was given in square centimetres, same measurement as the problem asked for. The difficulty of this item was much lower (70.5 %). The high success rate of the task was also contributed by the fact that the task was closed with a choice of answers.

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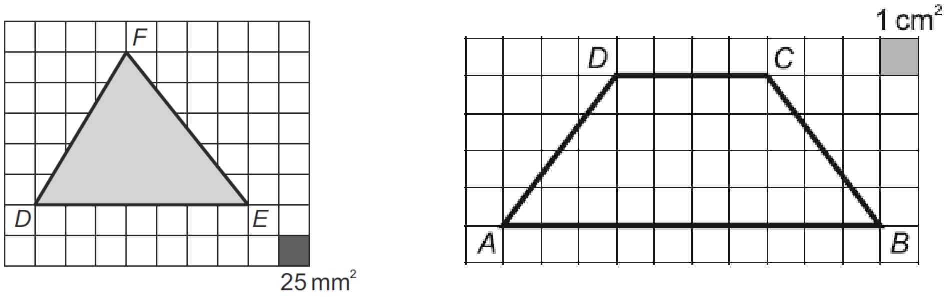
*Figure 7: a) Test item 7/2016: Each square in the grid has a surface area of 25 mm$^2$. Calculate the surface area of the triangle DEF in cm$^2$. Enter the result as a decimal number with accuracy on three decimal places.*
*b) Test item 20/2015: The surface area of the quadrilateral ABCD in the square grid is: A 22 cm$^2$, B 24 cm$^2$, C 28 cm$^2$, D 56 cm$^2$. (Source: NUCEM, 2016, 2015)*




A figure may complete the textual assignment as an *illustration* (or a sketch, a topological diagram), when it does not present the exact proportions (dimensions of angles, sides, ratio of lengths or perpendicularity) of the actual assignment; in these cases the pupil has to work not only with their theoretical knowledge, but also their imagination (Mesquita, 1998) – for example Fig. 10. We would like to point out that the figure as an illustration may present a big obstacle for some pupils. They may not realize that proportions are not preserved, and with such incorrect identification of relationships between objects they solve the problem. In other cases, the figure may be featured as an *object*, in which case it presents the proportions exactly and it is possible to discern other geometrical properties – e. g. Fig. 7a, b.

It is possible to view the role of figures in mathematical problems in another way as well (Mesquita, 1998). The figure has a *descriptive* role, if it only provides input data but offers no help in the solution itself. We can see such a figure in Fig. 9. A different role is the *heuristical* role, when the figure itself contains elements that instigate a correct solution. As an example, see Fig. 6a, b both of which come also as environments for solving the problem at hand. Divišová (2012) states, that differentiating between descriptive and heuristic figures is very subjective and is related to the pupil's "art of seeing".

*Figure 8: Figure as an object. Test item 15/2017: By drawing point F and two lines EF, FG, finish an isosceles trapezoid EFGH with bases EF and GH in a perpendicular coordinate system. What are the coordinates of point F, if point G has coordinates [2; - 1]? A [- 2; - 3] B [- 4; - 2] C [- 3; - 2] D [- 2; - 1] (Source: NUCEM, 2017a)*

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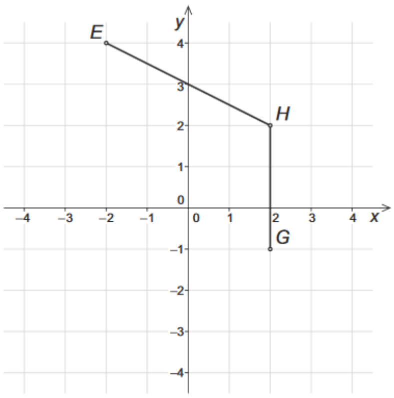
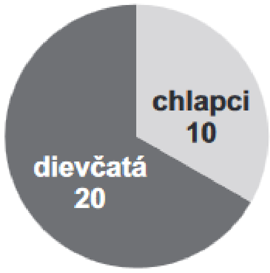



*Figure 9: Figure with a descriptive role. Test item 17/2016[9]: The bulletin board features a circular diagram showing the percentages of the members of the athletic team by sex. By how many degrees will the angle of the section of the circle displaying boys rise, if two more boys join the team and the number of girls will remain unchanged? A by 10°; B by 12°; C by 15°; D by 24°. (Source: NUCEM, 2016)*




As we stated earlier, years 2015 and 2016 were specific from the perspective of average success rate of pupils in solving test items featuring problems with figures compared to their success rate of solving test items in general (Fig. 2). The first interesting result is that the average success rate of solving test items in T9 testing was 53% in both years. The second one is that in 2015, the success rate of solving problems with figures was higher than general average (59 %) and in 2016 lower than general average by as much as 9 % (*i. e.* 44%). In 2016, the test contained nine problems with figures, two of which had a low success rate; these were presented in Fig. 7a and 9.

### 3. Conclusion

Visualisation of information and relationships between pieces of them belongs to basic cognitive strategies in creativity, discovery, resourcefulness and problem-solving. That visualisation in mathematics has its merit is indicated also by the fact that in international as well as national testing in mathematics periodically includes not only tasks, which can be solved using geometric strategies, but also "problems with figures". Quantitative, but also qualitative

[9] Success rate of this test item was 27.6 %.

analysis of the pupils' success rate in this type of tasks may help teachers of mathematics reveal the causes of failure and consequently effectively develop "the art of seeing" in primary and lower secondary school pupils. Graphical visualisation facilitates the imagining of a given phenomenon and shortens the learning process. Such a form of rising effectivity should be utilized in education as much as possible.

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